[surface reflection]
(Chapter 5.5.1 in *Elements*)

Reflection and transmission from a plane surface

We briefly review the essentials of reflection from a plane surface bounding an infinite medium, following Sears's development (1989) for infinitely thick materials. Figure 5-SR.1 shows the incident and refracted waves at a plane interface with a vacuum.

Specular reflection

If the incident and reflected wave vectors are $\vec{\mathbf{k}}_o$ and $\vec{\mathbf{k}}_1$ and $\hat{\vec{\mathbf{n}}}$ is the unit vector normal to the surface, then because the reflection is sp*ecular* (mirror-like),

$$\vec{\mathbf{k}}_o \cdot \hat{\vec{\mathbf{n}}} = -\vec{\mathbf{k}}_1 \cdot \hat{\vec{\mathbf{n}}} = \left(2\pi/\lambda\right) sin\left(\phi\right), \tag{5-SR1}$$

where ϕ is the incident grazing angle as in Fig. 21. The wave vector change in the reflection process is

$$\vec{\mathbf{q}} = \vec{\mathbf{k}}_1 - \vec{\mathbf{k}}_o = 2ksin\left(\phi\right)\hat{\vec{\mathbf{n}}} = \left(4\pi sin\left(\phi\right)/\lambda\right)\hat{\vec{\mathbf{n}}} \approx \left(4\pi\phi/\lambda\right)\hat{\vec{\mathbf{n}}}; \tag{5-SR2}$$

that is, the wave vector in reflectometry is always normal to the reflecting surface. Reflectometry probes the scattering or magnetization

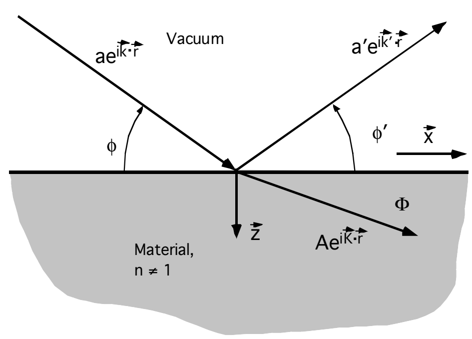

Figure 5-SR.1. Reflection and refraction at a plane interface between vacuum (air) and dense material.

The index of refraction is

$$n^2 = 1 - \xi \tag{5-SR3}$$

where, in the presence of a magnetic field of magnetization density $\vec{B}$,

$$\xi = \frac{\lambda^2}{\pi}\left(\rho b - \frac{2\pi m}{h^2}\vec{\mu} \cdot \vec{B}\right), \tag{5-SR4}$$

ρ is the atomic number density of scattering atoms, m is the neutron mass, λ is the neutron wavelength, $\vec{\mu}$ is the neutron magnetic moment and b is the coherent scattering length, so that

$$n = \sqrt{1 - \frac{\lambda^2}{\pi}\left(\rho b - \frac{2\pi m}{h^2}\vec{\mu}\cdot\vec{B}\right)} \approx 1 - \frac{\lambda^2}{2\pi}\left(\rho b - \frac{2\pi m}{h^2}\vec{\mu}\cdot\vec{B}\right). \quad (5\text{-SR5})$$

We continue without reference to the magnetic field interaction, but one can always replace

$$\rho b \to \rho b - \frac{2\pi m}{h^2}\vec{\mu}\cdot\vec{B} \quad (5\text{-SR6})$$

and account for neutrons of two polarization states.

The ratio of z-directed reflected and incident currents, Fig. 5.24, is the *reflectivity*,

$$R = \frac{|a'|^2}{|a|^2} = \frac{|1-n_z|^2}{|1+n_z|^2}, \quad (5\text{-SR7})$$

where

$$n_z = n\sin\phi . \quad (5\text{-SR8})$$

When the incident grazing angle ϕ is small, the reflected intensity relative to the incident intensity, the *reflectivity*, is

$$R = \left|1 - \left[1-(\varphi_c/\varphi)^2\right]^{\frac{1}{2}}\right|^2 \Big/ \left|1 + \left[1-(\varphi_c/\varphi)^2\right]^{\frac{1}{2}}\right|^2, \quad (5\text{-SR9})$$

in which ϕ is the incident grazing angle and the *critical angle* is

$$\phi_c = \lambda\sqrt{\frac{\rho b}{\pi}}, \quad (5\text{-SR10})$$

and the critical wave vector is

$$q_c = 4\pi\varphi_c/\lambda . \quad (5\text{-SR11})$$

When $\varphi < \varphi_c$, the radicand $1-(\varphi_c/\varphi)^2 < 0$ is negative and $\left[1-(\varphi_c/\varphi)^2\right]^{\frac{1}{2}}$ is purely imaginary so that the numerator and denominator are complex conjugates of identical magnitude. Then $R = 1.0$, reflection is perfect, 100%, and the transmission is zero. This mathematical step is the basis for plateaus that arise in neutron wave propagation theory.

When $\phi > \varphi_c$, the radicand is a positive real number, the numerator is less than the denominator, and $R < 1$. Figure 5-SR.1 shows the reflectivity as a function of the ratio φ_c/φ. In the limit of large φ,

$$R \to 1/16\left(\frac{\varphi}{\phi_c}\right)^{-4}. \tag{5-SR12}$$

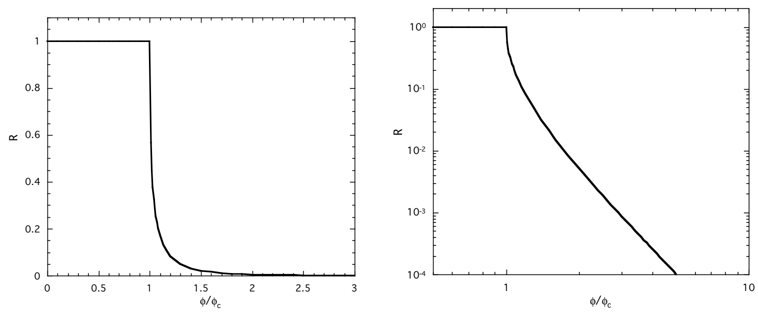

Figure 5-SR.1. Reflectivity as a function of grazing angle, normalized to the critical angle, for real scattering length density. Left, linear scales. Right, logarithmic scales.

This example illustrates the presence of a plateau within which the reflectivity is 100%, which is a universal feature of surface reflectivity when the scattering lengths are real numbers (no absorption). The critical angle is the angle below which the surface reflectivity is perfect, 100%, because the magnitudes of the numerator and denominator in (5.28) in *Elements* are equal. This feature represents an absolute scale marker for the measured reflectivity data.

References

Sears, V. F. (1989). *Neutron Optics*. New York: Oxford University Press.