[spin precession methods]
(Chapter 5.4.2 in *Elements*)

Spin precession methods

Hughes, Wallace and Holzmann were the first to observe neutron polarization (Hughes et al. 1948). In 1972, Ferenc Mezei first introduced the *neutron spin-echo* (NSE) technique as a method of spectroscopy, that is, measurement of probabilities of energy transfer in sample materials (Mezei 1972). The basis of NSE and its related elaborations is the precession of neutron spins in magnetic fields. The neutron spin revolves (*precesses*) around magnetic fields $\vec{B}$ at the *Larmor frequency*

$$\omega_L = \gamma\left|\vec{B}\right| , \qquad (5\text{-SP1})$$

where $\gamma = 2\mu/\hbar = 1.832\times10^8\ radians/s - T$ is the *Larmor constant*, or the *gyromagnetic ratio* and μ is the magnitude of the neutron magnetic moment, $\mu = \left|\vec{\mu}\right| = 6.03331\times10^{-8}\,\frac{eV}{T}$. The magnetic moment $\vec{\mu}$ is collinear with but opposite to the direction of the spin, $\vec{S}$,

$$\vec{\mu} = -\gamma\vec{S} . \qquad (5\text{-SP2})$$

(Here we take γ to represent the absolute value of the gyromagnetic ratio—sometimes writers use the signed, negative value.) The magnitude of the spin is $\left|\vec{S}\right| = \frac{1}{2}\hbar$. The motion of the spin is according to Bloch's equation,

$$\frac{d\vec{S}}{dt} = -\gamma\vec{S}\otimes\vec{B} = \gamma\vec{B}\otimes\vec{S} , \qquad (5\text{-SP3})$$

which says that the rate of change of the spin vector lies perpendicular to the magnetic field and perpendicular to the spin vector. The component of the spin perpendicular to $\vec{B}$ revolves about the field with angular frequency ω_L, consistent with the right-hand rule and the sign in Eq. 5-SP3, counter-clockwise when looking into the $\vec{B}$-vector, as in Fig. 5.8.[#]

[#] The $\vec{B}$-vector on the axis of a circular current loop is perpendicular to the plane of the current loop and in the direction of the right-hand thumb when the current is in the direction of the fingers. The $\vec{B}$-vector is in the outward direction from the north pole of a permanent bar-magnet.

For example, a neutron spin revolves about a 100-gauss (.01 T) field at angular frequency $\omega_L = 1.832 \times 10^6\ radian/s$ (cyclic frequency $f_L = \frac{1}{2\pi}\omega_L = 292.\ kHz$). For another example, the spin of a 1000-m/s neutron (4-Å) traveling 1 cm in a 34-gauss field revolves through an angle

$$\theta = \gamma \left| \vec{B} \right| \frac{\ell}{\mathrm{v}} = (1.832 \times 10^8)(34.)(0.01)/1000. = 2\pi\ , \qquad (5\text{-SP4})$$

that is, one turn, where ℓ is the distance traveled and v is the neutron speed. Whatever is the initial angle ϕ between the spin and the magnetic field, that angle remains unchanged during rotation.

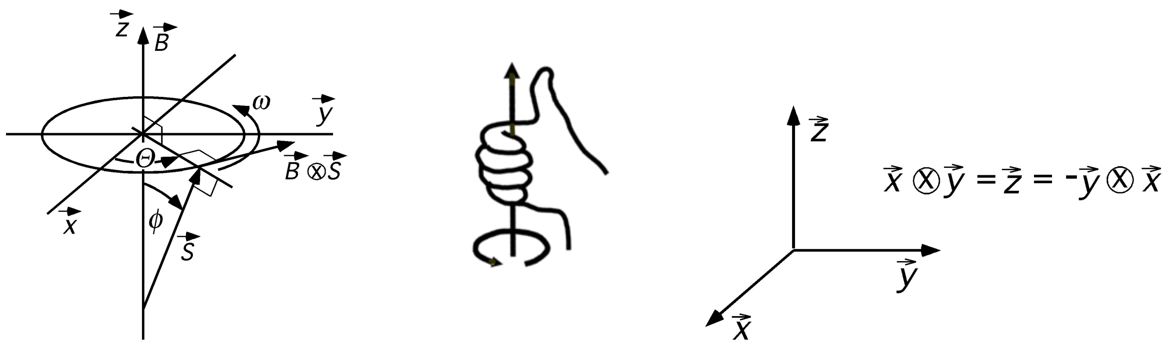

Figure 5-SP1. Left. A neutron precessing in a magnetic field. The magnetic moment $\vec{\mu}$ *is collinear with but opposite to the direction of the* spin, $\vec{S}$. *Right. The right-hand rule and a right-handed coordinate system.*

There is no energy associated with the spin precession as such, but the energy of the neutron in the field is

$$V = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi\ . \qquad (5\text{-SP5})$$

The precession frequency is always ω_L, independently of ϕ, and dependent only on the magnitude of B.

The concept of *polarization* of a population of neutrons is ubiquitous in discussion of spin-precession methods. If there are $N_\uparrow$ neutrons with spins parallel to a reference direction and $N_\downarrow$ with spins antiparallel the polarization of the populations is

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}. \qquad (5\text{-SP6})$$

In a neutron beam with currents $J_\uparrow$ and $J_\downarrow$, the polarization is the same:

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = \frac{vN_\uparrow - vN_\downarrow}{vN_\uparrow + vN_\downarrow} = \frac{J_\uparrow - J_\downarrow}{J_\uparrow + J_\downarrow}. \quad (5\text{-SP8})$$

Polarizations are, in general, wavelength dependent, of course.

<u>SESANS</u>

DO MORE HERE TO RELATE THE DIAGRAM TO CALCULATIONS THAT FOLLOW

Theo Rekveldt has adapted the spin-echo method to small-angle diffraction measurements, *spin-echo SANS* (Rekveldt et al. 2003) SESANS allows measurement of long-range density-density correlations without the severe losses in intensity. Figure 5-SE2 illustrates the layout of a spin-echo small-angle diffractometer.

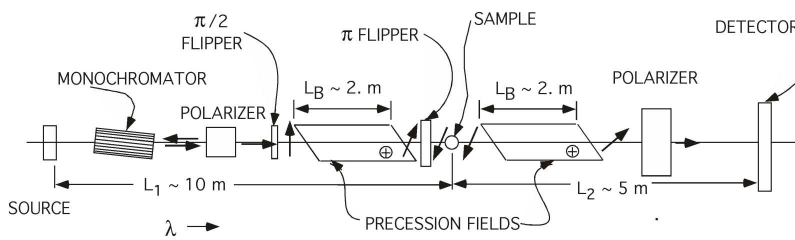

Figure 5-SP2. Conceptual arrangement of an SESANS instrument.

The advantage of spin-echo methods is that the wavelength change or the angular deflection is measured with high precision without requiring narrow definition of wavelengths or angles. Moreover, spin-echo methods apply to neutron beams of large area. Relaxed wavelength selection and relaxed spatial and angular collimation, which decouple intensity-related factors from the quantity to be resolved, allow for precise measurements with high instrument throughput, with sensitivity beyond what is accomplishable using monochromatic beam methods.

To scan a range of *r*s requires scanning a range of field strengths B and/or a range of wavelengths λ. In the reactor implementation a drum monochromator selects a band of wavelengths $\Delta\lambda/\lambda \sim 10\%$ around the nominal wavelength λ, and r varies with B for the selected wavelength. In the time-of-flight version, r varies with wavelength at fixed B, and a different range of r s follows for different B s. There is no monochromator, polarizer and flippers must be wavelength-tuned or white-beam devices, and the wavelength is related to the time of arrival of neutrons

$$t = L_{total}/v = L_{total}(m/h)\lambda;\ \lambda = (h/m)\frac{t}{L_{total}}. \quad (5\text{-SP9})$$

Here L_{total} is the total length of the flight path from the source to the detector and time t is measured from the time of the source pulse. In terms of round numbers and representative parameters, $B_{max} = 6000$ gauss, $L = 2\ m$, $\lambda = 4\ \AA$, $\psi = 30°$, and the maximum Fourier length is $r_{max} = 3.9 \times 10^4\ \AA = 3.9\ \mu m$. The corresponding Q-resolution, calculated as $\Delta Q = 2\pi / r_{max}$, is $\Delta Q = 1.6 \times 10^{-4}\ \AA^{-1}$. This can be compared to the minimum Q in a conventional SANS instrument, $Q_{min} \sim 1.0 \times 10^{-3}\ \AA^{-1}$, although SESANS measures a slit-smeared image in the direct position space, while SANS with aperture collimation measures in the Fourier transform, i.e., wave vector space. There are numerous variations on this theme, described in Rekveldt 2003.

Figure 5-SP3. The OFFSPEC spin echo SANS instrument at ISIS TS-2. Typical data (Image STFC/Steven Kill).

References

Hughes, D. J., J. R. Wallace, and R. W. Holzman (1948). Neutron polarization. *Phys. Rev.* **73**, 1277.

Mezei, F. (1972). Neutron spin echo: a new concept in polarized thermal neutron techniques. *Z. Physik* **255**, 146-60.

Rekveldt, M. T. W., W. G. Bouman, W. H. Kraan, O. Uca, S. V. Grigoriev, R. Krueger (2003). Elastic neutron scattering measurements using Larmor precession of polarized neutrons. In *Neutron Spin Echo Spectroscopy*, ed. F. Mezei, C. Pappas, and T. Gutberlet. *Lecture Notes in Physics*. Berlin: Springer, pp. 87-99.