[reciprocal lattice]
(Chapter 5.3 in *Elements*)

Reciprocal Lattice and Reciprocal Lattice Vectors

Reciprocal lattice vectors are 3-D vectors $\vec{\mathbf{k}}_i$ such that

$$exp(i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})=1\,, \qquad \text{(5-RL1)}$$

where $\vec{\mathbf{r}}$ is the location of any (i.e., all) of the direct crystal lattice points,

$$\vec{\mathbf{r}}_{\ell mn}=\ell\vec{\mathbf{b}}_1+m\vec{\mathbf{b}}_2+n\vec{\mathbf{b}}_3 \qquad \text{(5-RL2)}$$

where ℓ, m, n are integers and $\vec{\mathbf{b}}_i$ are the (non coplanar) direct lattice basis vectors. Integer combinations $\vec{\mathbf{Q}}_{pqr}$ of vectors $\vec{\mathbf{a}}_i=2\pi\frac{\vec{\mathbf{b}}_j\otimes\vec{\mathbf{b}}_k}{V}$ $(i,j,k\ in\ cyclic\ order)$, where $V=\vec{\mathbf{b}}_i\cdot\vec{\mathbf{b}}_j\otimes\vec{\mathbf{b}}_k$ is the volume of the direct lattice cell, satisfy the condition (RL1). The set of all such vectors $\vec{\mathbf{Q}}_{pqr}=p\vec{\mathbf{k}}_i+q\vec{\mathbf{k}}_j+r\vec{\mathbf{k}}_k\,(integers\ p,q,r)$ then form a lattice, which is in one-to-one relation to the direct lattice, because

$$\vec{\mathbf{b}}_i=\frac{1}{2\pi}\vec{\mathbf{k}}_j\otimes\vec{\mathbf{k}}_k\,. \qquad \text{(5-RL3)}$$

The $\vec{\mathbf{Q}}_{pqr}$ -lattice is called the *reciprocal lattice* and the $\vec{\mathbf{a}}_i$ s are the *basis* of the reciprocal lattice.