[filter-difference spectrometer resolution]
(Chapter 6.2.5 in *Elements*)

Resolution of Filter-Difference Spectrometers

The energy-transfer resolution is the energy-transfer width inferred from the width of the observed time distribution, which is attributed to $E_i(t)$ through t_e and t_d and E_f which are assumed to be statistically independent,

$$\delta E_i(t) = \left| \frac{\partial E_i(t)}{\partial t} \right| |\delta t| . \tag{6-FDS-1}$$

The following treatment ignores variations in flight path lengths and correlations among these and other unmentioned variables. Ignoring these correlations, their variances add independently into the variance of *t*, so that from *Elements* Eq. 6.20,

$$(\delta t)^2 = (\delta t_e)^2 + (\delta t_d)^2 + \frac{m}{2}\left[\frac{\partial}{\partial E_f}\left(\frac{L_i}{\sqrt{E_f+\varepsilon}} + \frac{L_f}{\sqrt{E_f}} \right)\right]^2 (\delta E_f)^2 =$$

$$= (\delta t_e)^2 + (\delta t_d)^2 + \frac{m}{8}\left(\frac{L_i}{E_i^{3/2}} + \frac{L_f}{E_f^{3/2}} \right)^2 (\delta E_f)^2 . \tag{6-FDS-2}$$

Because $\overline{E_f}$ in *Elements* Eq. 6.19 is a mean value, not statistically distributed,

$$(\delta\varepsilon)^2 = \left(\frac{\partial E_i(t)}{\partial t} \right)^2 (\delta t)^2 \tag{6-FDS-3}$$

and

$$\left(\frac{\partial E_i(t)}{\partial t} \right)^2 = \left(\frac{2E_i(t)}{t - \left(\overline{t_e} + \overline{t_f} + \overline{t_d}\right)} \right)^2 = \frac{8}{m}\frac{E_i^3(t)}{L_i^2}, \tag{6-FDS-4}$$

$$(\delta\varepsilon)^2 = \frac{8}{m}\frac{E_i^3(t)}{L_i^2}\left[(\delta t_e)^2 + (\delta t_d)^2 + \frac{m}{8}\left(\frac{L_i}{E_i^{3/2}} + \frac{L_f}{E_f^{3/2}} \right)^2 (\delta E_f)^2 \right]. \tag{6-FDS-5}$$

This can be written

$$\frac{(\delta\varepsilon)^2}{E_i^2} = \left[4\frac{v_i^2}{L_i^2}\left((\delta t_e)^2 + (\delta t_d)^2 \right) + \frac{1}{E_i^2}\left(1 + \frac{L_f}{L_i}\left(\frac{E_i}{E_f} \right)^{3/2} \right)^2 (\delta E_f)^2 \right], \tag{6-FDS-6}$$

which is identical to Eccleston's expression for the crystal analyzer case in which

$$\left(\delta E_f\right)^2 = 4E_f^2 \cot^2\theta\left(\delta\theta\right)^2. \qquad \text{(6-FDS-7)}$$

One notes that δt_e and δt_d are small and roughly inversely proportional to v.

For the filter-difference spectrometer case the width of the accepted energy band is $E_{cut-off}^{Be} - E_{cut-off}^{BeO} = 1.54\ meV$, the mean scattered neutron energy is $\overline{E_f} = 4.35 meV$, and the variance of the accepted energy band is

$$\left(\delta E_f\right)^2 = \frac{1}{12}\left(E_{cut-off}^{Be} - E_{cut-off}^{BeO}\right)^2 = 0.198\ meV. \qquad \text{(6-FDS-8)}$$

Reference

Eccleston, R. J. (2003). Time-of-flight inelastic neutron scattering. In *Neutron Data Booklet*, 2nd ed., ed. A. Dianoux and G. Lander. Grenoble, France: Institut Laue-Langevin, Chap. 2.6.