[cylindrical detector efficiency]
(Chapter 8.2.1 in *Elements*)

Efficiency of a cylindrical detector

Assuming a uniform incoming neutron beam and averaging over one-half of the detector diameter, the efficiency is

$$\varepsilon(\lambda) = \frac{1}{R}\int_0^R dz \int_{y_{\min}}^{y_{\max}} \exp\left(-N\sigma\left(y - y_{\min}\right)\right)dy, \qquad \text{(8-CDE-1)}$$

in which z is measured perpendicular to the axis of the detector and R is the radius of the cylinder, as in Figure 8-CDE-1.

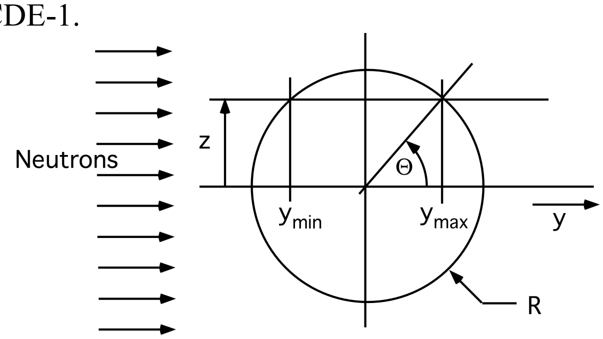

Figure 8-CDE-1 Neutron paths in a cylindrical detector.

Taking $z = R\sin(\theta)$, $y_{\min} = -R\cos(\theta)$, and $y_{\max} = Rcos(\theta)$, the expression becomes

$$\varepsilon(\lambda) = 1 - \int_0^{\frac{\pi}{2}} \exp\left(-2N\sigma R\cos(\theta)\right)\cos(\theta)d\theta. \qquad \text{(8-CDE-1)}$$

A useful series representation results from expanding the exponential in a power series,

$$\varepsilon(\lambda) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}X^n}{n!}Z_n, \qquad \text{(8-CDE-2)}$$

where

$$X = 2N\sigma(\lambda)R \qquad \text{(8-CDE-3)}$$

and

$$Z_n = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n}{2}+\frac{3}{2}\right)} = \begin{cases} \frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{32},\ldots \text{ for } n = 1,3,5,\ldots = 2p+1; \\ \frac{2}{3},\frac{8}{15},\frac{48}{105},\ldots \text{ for } n = 2,4,6,\ldots = 2p \end{cases} . \quad \text{(8CDE-4)}$$

The coefficients are easily generated recursively from $Z_o = 1$ and $Z_1 = \frac{\pi}{4}$,

$$Z_n = \frac{p+1/2}{p+1} Z_{n-2} \text{ for } n = 1,3,5,\ldots = 2p+1 \text{ and} \quad \text{(8-CDE-5)}$$

$$Z_n = \frac{p}{p+1/2} Z_{n-2} \text{ for } n = 2,4,6,\ldots = 2p . \quad \text{(8-CDE-6)}$$

The series converges for all *X*. When the magnitude of successive terms is diminishing with increasing *N*, the absolute error after summing *N* terms is

$$|\Delta\varepsilon| \leq \frac{X^{N+!}}{(N+1)!} Z_{N+1} . \quad \text{(8-CDE-7)}$$

The limits for large and small *X* can be inferred from inspection of the starting expression,

$$\lim_{X\to 0} |\Delta\varepsilon(X)| = \pi \frac{X}{4} \text{ and } \lim_{X\to \boldsymbol{large}} |\Delta\varepsilon(X)| = 1 . \quad \text{(8-CDE-8)}$$

A method for rapid calculation of CDEindrical counter efficiency, Carpenter 1981, based on a series expansion of a correction of the first-order result, , a mean-chord approximation.

$$\varepsilon(\lambda) \approx 1 - \exp\left(-\frac{\pi}{4}X\right) . \quad \text{(8-CDE-9)}$$

The full series expression is

$$1-\varepsilon(\lambda) = \exp\left(-\frac{\pi}{4}X\right)\left(\sum_{m=0}^{M}\left(\frac{\pi}{4}X\right)^m Y_m + R_M\right), \quad \text{(8-CDE-10)}$$

where

$$Y_m = \sum_{n=0}^{m} \frac{1}{n!(m-n)!} \left(-\frac{\pi}{4} \right)^n Z_n , \quad \text{(8-CDE-11)}$$

in which the Z_n s are as above. The Z_n and Y_m are independent of X, therefore, independent of wavelength and geometry, that is, universal for all detector sizes and all wavelengths, and are tabulated in IPNS Note 17. The remainder after M terms is R_M, which is bounded by

$$R_M \le \left(\frac{\pi}{4} X \right)^{M+1} \frac{Y_{M+1}}{1-r_M} , \quad \text{(8-CDE-12)}$$

in which

$$r_m = \left(\frac{\pi}{4} X \right) \frac{Y_{m+1}}{Y_m} \quad \text{(8-CDE-13)}$$

is the ratio of successive terms in the Y-expansion.

The mean position of detection, $\bar{y}$, measured from the cylindrical axis, is

$$\bar{y} = \frac{1}{N\sigma(\lambda)} + \frac{\pi R}{4} \left[1 + \frac{\xi - 2}{\varepsilon(\lambda)} \right], \quad \text{(8-CDE-14)}$$

where

$$\xi = \sum_{m=1}^{\infty} m \left(\frac{X\pi}{4} \right)^{m-1} Y_m . \quad \text{(8-CDE-15)}$$

Therefore the mean time of detection, measured from the time of crossing the detector axis, is

$$\bar{t} = \frac{\bar{y}}{\mathrm{v}} , \quad \text{(8-CDE-16)}$$

where v is the neutron speed. Always, $\bar{y}$ and $\bar{t}$, are less than zero.

Reference

Carpenter, J. M. (1981). Efficient code for calculating cylindrical gas proportional counter efficiency, unpublished technical note IPNS Note #17.