[chopper counting rate and resolution]
(Chapter 6.1.2 in *Elements*)

**Counting Rate**

The time-average rate of neutrons scattered into a detector pixel ad registered (counted), that is, the counting rate, is

$$R\left(\lambda_o,\lambda,\vec{\Omega}\right)=N_{sample}\frac{\partial^2\sigma}{\partial\lambda\partial\Omega}\left(\lambda_o,\vec{\Omega}_o\rightarrow\lambda,\vec{\Omega}\right)\Phi\left(\lambda_o\right)\eta\left(\lambda\right)\Delta\Omega\Delta\lambda\,, \tag{6-CCR-1}$$

where $N_{sample}$ is the number of scattering units illuminated by the incident beam,

$$N_{sample}=A_{sample}nd_{sample}\,, \tag{6-CCR-2}$$

$A_{sample}$ is the area of the sample illuminated by the neutron beam as seen from the beam direction, $n$ is the number density of scattering units in the sample, and $d_{sample}$ is the thickness of the sample measured in the beam direction. The differential scattering cross section per scattering unit for scattering from $\lambda_o,\vec{\Omega}_o$ into $\lambda,\vec{\Omega}$, per unit scattered neutron wavelength and per unit scattered neutron solid angle, is

$$\frac{\partial^2\sigma}{\partial\lambda\partial\Omega}\left(\lambda_o,\vec{\Omega}_o\rightarrow\lambda,\vec{\Omega}\right). \tag{6-CCR-3}$$

The efficiency of the detector pixel is $\eta\left(\lambda\right)$, and the solid angle subtended by the detector pixel around direction $\vec{\Omega}$ is

$$\Delta\Omega=\frac{A_{pixel}}{L_3^2}\,, \tag{6-CCR-4}$$

where $A_{pixel}$ is the area of the detector pixel as seen from the sample. The wavelength interval into which scattered neutrons are sorted is

$$\Delta\lambda=\left(\frac{h}{m}\right)\frac{\Delta t_d}{L_3}\,, \tag{6-CCR-5}$$

in which $\Delta t_d$ is the detector time-of-arrival interval into which scattered neutron events are accumulated.

**Resolution**

The contours of uncertainty with which the instrument samples $\vec{Q}$ and $\Delta E$ are ellipsoids in the four-dimensional $\vec{Q},\Delta E$ space. It is beyond the scope of the present discussion to explore this subject in depth. However, we can estimate the energy-transfer resolution rather simplistically.

For illustration, we ignore the primary source-pulse contribution and the detector time delay, which are small. The main contributors to the energy transfer resolution are the moderator emission time distribution and the $E_o$ chopper open-time distribution. These are independently distributed quantities; consequently, it is correct to express the variance of the energy transfer distribution $(\delta\Delta E)^2$ as the sum of variance contributions due to the emission time $\delta t_e$ and chopper opening time $\delta t_{ch}$:

$$(\delta\Delta E)^2 = \left(\frac{\partial\Delta E}{\partial t_e}\right)^2 (\delta t_e)^2 + \left(\frac{\partial\Delta E}{\partial t_{ch}}\right)^2 (\delta t_{ch})^2, \qquad \text{(6-CCR-6)}$$

where $\delta t_{ch}$ is the standard deviation of the chopper open-time distribution. (see this website [Fermi choppers] and also *Elements* Chap. 7.4.2, 7.4.3—disk choppers—and 7.4.4). The standard deviation of the emission time distribution is $(\delta t_e)$ (see *Elements* Chap. 2.3.2) and the cautionary note with Loong, et al. 1993. Then

$$\frac{\partial\Delta E}{\partial t_e} = \left(\frac{\partial E_o}{\partial t_e} - \frac{\partial E}{\partial t_e}\right) \text{ and } \frac{\partial\Delta E}{\partial t_{ch}} = \left(\frac{\partial E_o}{\partial t_{ch}} - \frac{\partial E}{\partial t_{ch}}\right). \qquad \text{(6-CCR-7)}$$

Referring to (6-CCR-1,5,6, and 7) we find

$$\frac{\partial\Delta E}{\partial t_e} = -\frac{\partial\Delta E}{\partial t_{ch}} = 2\left(\frac{h}{m}\right)\left[\frac{E_o}{\lambda_o L_1} + \frac{E}{\lambda L_3}\frac{L_2}{L_1+L_2}\right], \qquad \text{(6-CCR-8)}$$

so that the variance of the energy transfer distribution, noting that $\lambda = \frac{h}{mv}$, is

$$(\delta\Delta E)^2 = 4\left[\frac{v_o E_o}{L_1} + \frac{vE}{L_3}\frac{L_2}{L_1+L_2}\right]^2 \left((\delta t_e)^2 + (\delta t_{ch})^2\right). \qquad \text{(6-CCR-9)}$$

For elastic scattering, $E = E_o$, this gives

$$\delta\Delta E = 2E_o v_o\left[\frac{1}{L_1} + \frac{1}{L_3}\frac{L_2}{L_1+L_2}\right]\sqrt{(\delta t_e)^2 + (\delta t_{ch})^2}\ . \qquad \text{(6-CCR-10)}$$

For values representative of no particular spectrometer, $E_o = 10\ meV$, $v_o = 1383\ m/s$, $L_1 = 10\ m$, $L_2 = 1\ m$, $L_3 = 4\ m$, $\delta t_e = 50 / 2.35 = 21.3\ \mu s$, and $\delta t_{ch} = 25 / 2.35 = 10.6\ \mu s$. This comes to

$$\delta\Delta E = 0.0807\ meV \tag{6-CCR-11}$$

for the variance of the energy transfer. If all the distributions have Gaussian form, which is only vaguely true at best, the full width at half maximum of the energy transfer resolution for elastic scattering, an often-quoted characterization of a chopper spectrometer, is

$$fwhm_{\Delta E} = 2.35\ \delta\Delta E = 0.190 meV \tag{6-CCR-12}$$

and for the relative elastic scattering resolution,

$$\frac{fwhm_{\Delta E}}{E_o} = 0.0190 = 1.9\%. \tag{6-CCR-13}$$

**Reference**


C. K. Loong, J. M. Carpenter, and S. Ikeda (1993). A parametric formulation of the resolution function of a pulsed-source chopper spectrometer. In *Proc. XII$^{th}$ Mtg. Int. Collaboration Advanced Neutron Sources*, no. 94-025, vol. 1. Oxford: Rutherford-Appleton Laboratory, pp. I-320-25.

Loong, C.-K., Carpenter, J. M., and Ikeda, S. (1987). Resolution function of a pulsed-source chopper spectrometer. *Nucl. Instr. & Meth.* A, **260**, 381.


Caution: an important typographical error in these papers needs to be corrected. The $t^2$ term in the expression for the moderator emission-time distribution is $(a-b)^2 t^2$, not $(a+b)^2 t^2$.