[Top-Hat Broadening]
(Chapter 2.3.2 in *Elements*)

Top-hat broadening, or, long-pulse-source broadening

When the source pulse is extended in time, as in long-pulse sources, the finite duration of the source pulse broadens the emission time distribution from its form for a delta function source pulse. The same effect occurs when the observed (or simulated) time distribution includes the variable time delay due to acceptance of a range of wavelengths. Representing the source pulse as a *step function* (*Heaviside function*) time distribution,

$$H(t) = \frac{1}{T}\begin{cases} 1 \text{ for } 0 < t < T \text{ and} \\ 0 \text{ for } t < 0 \text{ and for } t > T, \end{cases} \tag{1}$$

the broadened Ikeda-Carpenter (I-C) function is

$$i_H(t) = \int_0^t i(t-\tau)H(\tau)d\tau = \frac{1}{T}\int_{\tau_{\min}}^t i(t-\tau)d\tau, \tag{2}$$

where $\tau_{\min} = Max(0, t-T)$. Explicitly, collecting terms of equal order in τ, we have

$$i_H(t) = \frac{2}{aT}\int_{\tau_{\min}}^t \left\{\left[(1-R)a^2 - \frac{Ra\beta^2}{(a-\beta)}\right]\tau^2 \exp(-a\tau) - \right.$$
$$\left. - \frac{2Ra\beta^2}{(a-\beta)^2}\tau\exp(-a\tau) + \frac{2Ra\beta^2}{(a-\beta)^3}\left(\exp(-\beta\tau) - \exp(-a\tau)\right)\right\}d\tau \cdot \tag{3}$$

Defining functions h_o, h_1, and h_2, which are standard integrals found in the handbooks,

$$h_o(x,t) = x\int_{\tau_{\min}}^t \exp(-x\tau)d\tau = \left(\exp(-x\tau_{\min}) - \exp(-xt)\right), \tag{4}$$

$$h_1(x,t) = x^2\int_{\tau_{\min}}^t \tau\exp(-x\tau)d\tau =$$
$$= \left[(x\tau_{\min}+1)\exp(-x\tau_{\min}) - (xt+1)\exp(-xt)\right], \text{ and} \tag{5}$$

$$h_2(x,t) = x^3\int_{\tau_{\min}}^t \tau^2\exp(-x\tau)d\tau =$$
$$= \left[(x^2\tau_{\min}^2 + 2x\tau_{\min} + 2)\exp(-x\tau_{\min}) - (x^2t^2 + 2xt + 2)\exp(-xt)\right], \tag{6}$$

we finally have

$$i_H(t) = \frac{a}{2T}\left\{\left[(1-R)a^2 - \frac{Ra\beta^2}{(a-\beta)^3}\right]\frac{h_2(a,t)}{a^2} - \frac{2Ra\beta^2}{(a-\beta)^2}\frac{h_1(a,t)}{a} + \right.$$

$$+ \frac{2Ra\beta^2}{(a-\beta)^2}\left(\frac{h_o(a,t)}{a} - \frac{h_1(\beta,t)}{\beta}\right)\Bigg\} \quad . \tag{7}$$

Recalling that $\tau_{min} = Max(0, t - T)$, there are three regimes: $0 < t < T$, where $\tau_{min} = 0$, $t > T$, where $\tau_{min} = t - T$, and $t < 0$, where $i_w(t) = 0$.

The mean emission time is

$$t_e = \frac{3}{a} + \frac{R}{\beta} + \frac{T}{2}, \tag{8}$$

and the variance of the emission time distribution is

$$\sigma^2 = \frac{3}{a^2} + \frac{(2R - R^2)}{\beta^2} + \frac{T^2}{12}. \tag{9}$$

These results will be useful for fitting measured and calculated data and for instrument design simulations.

Readers interested to derive broadened I-C functions for other forms of sectionally continuous broadening may find useful the very general function-transform pair for rational functions (ratios of polynomials in the transform variable) found, for example, in the Bateman tables (*Tables of Integral Transforms* 1954).