[Time Dependence]
(Chapter 1.10 of *Elements*)
Because this is an initial-value problem, it is appropriate to invoke the Laplace transform, in which s is the time-related Laplace transform variable,

$$\tilde{\varphi}(E,s) = \mathscr{L}\left[\varphi(E,t)\right] = \int_0^\infty \exp(-st)\varphi(E,t)\,dt \tag{1}$$

and

$$\mathscr{L}\left[\frac{\partial\varphi(E,t)}{\partial t}\right] = s\tilde{\varphi}(E,s) - \varphi(E,t=0)\cdot \tag{2}$$

Assuming that the initial flux is zero and that the source is a delta function at time $t = 0$,

$$\mathscr{L}\left[S(E,t)\right] = \mathscr{L}\left[S(E)\delta(t)\right] = S(E)\,. \tag{3}$$

The transformed diffusion equation is

$$\tfrac{1}{v}s\tilde{\varphi}(E,s) = -\Sigma_{scatt}(E)\tilde{\varphi}(E,s) + \int_E^\infty dE'\frac{\Sigma_{scatt}(E')}{E'}\tilde{\varphi}(E',s) + S(E)\ . \tag{4}$$

We already have the solution of this equation. Taking the result for a specific source energy and replacing the absorption cross section $\Sigma_{abs}(E) \rightarrow \frac{s}{v(E)}$ so that

$$\Sigma_{total}(E) \rightarrow \Sigma_{scatt}(E) + \frac{s}{v(E)}\,, \tag{5}$$

we obtain

$$\tilde{\varphi}(E,s) = \frac{S_o\delta(E - E_Q)}{\Sigma_{scatt}(E_Q) + \frac{s}{v_Q}} + \frac{S_o}{E_Q}\frac{\Sigma_{scatt}(E_Q)U(E < E_Q)}{\left(\Sigma_{scatt}(E_Q) + \frac{s}{v_Q}\right)\left(\Sigma_{scatt}(E) + \frac{s}{v}\right)}\ \exp\left(\int_E^{E_Q}\frac{\Sigma_{scatt}(E')}{\Sigma_{scatt}(E') + \frac{s}{v'}}\frac{dE'}{E'}\right). \tag{6}$$

Here, $v_Q = v(E_Q)$ and $v' = v(E')$. Although s is a complex number, it is independent of $E,$ and if we assume that $\Sigma_{scatt}(E')$ is constant, as is reasonable for epithermal energies, the integral is do-able,

$$\int_E^{E_Q} \frac{\Sigma_{scatt}}{\Sigma_{scatt} + \frac{s}{v'}} \frac{dE'}{E'} = 2\ln\left(\frac{s + v_Q\Sigma_{scatt}}{s + v\Sigma_{scatt}}\right) \tag{7}$$
,

so

$$\exp\left(-\int_E^{E_Q} \frac{\Sigma_{scatt}}{\Sigma_{scatt} + \frac{s}{v'}} dE'\right) = \left(\frac{s + v_Q\Sigma_{scatt}}{s + v\Sigma_{scatt}}\right)^2 \tag{8}$$
,

and we have the simple result

$$\tilde{\varphi}(E,s) = \frac{v_Q S_o \delta(E - E_Q)}{s + v_Q\Sigma_{scatt}} + \frac{v_Q S_o}{E_Q} \frac{v\Sigma_{scatt}(s + v_Q\Sigma_{scatt})}{(s + v\Sigma_{scatt})^3} U(E < E_Q). \tag{9}$$

The singularities of $\tilde{\varphi}(E,s)$ in the complex s-plane determine the form of the inverse transform (consult a table of Laplace transform/function pairs):

$$\mathscr{L}^{-1}[1] = \delta(t) \quad , \tag{10}$$

$$\mathscr{L}^{-1}\left[\frac{1}{s+a}\right] = \exp(-at) \ , \tag{11}$$

and

$$\mathscr{L}^{-1}\left[\frac{f(s)}{(s+a)^3}\right] = \frac{1}{2} f(-a) t^2 \exp(-at), \tag{12}$$

so we have

$$\varphi(E,t) = v_Q S_o \exp(-v_Q\Sigma_{scatt}t)\delta(E - E_Q) + \frac{1}{2}\frac{v_Q S_o}{E_Q}(v_Q - v)v(\Sigma_{scatt}t)^2 \exp(-v\Sigma_{scatt}t) \quad U(E < E_Q). \tag{13}$$

Source neutrons at energy E_Q disappear rapidly with a time constant $\frac{1}{v_Q\Sigma_{scatt}}$. Neglecting these, noting that $\frac{v_Q^2}{E_Q} = \frac{v^2}{E}$ and approximating $(v_Q - v) \approx v_Q$ when $v << v_Q$, the final result is the *slowing-down time distribution* for hydrogenous moderators,

$$\varphi(E,t) = \frac{1}{2}\frac{S_o}{E}(v\Sigma_{scatt}t)^2 \exp(-v\Sigma_{scatt}t) U(E < E_Q). \tag{14}$$

This slowing-down time distribution is another part of the basis for the I-C pulse shape function (described in Chapter 2 of *Elements*).

It is easy to obtain the *time-integrated flux* $\varphi(E) = \int_0^\infty \varphi(E,t)dt$ from the Laplace transform:

$$\varphi(E) = \tilde{\varphi}(E,s)\Big|_{s=0} = \frac{1}{E}\frac{S_o}{\Sigma_{scatt}} U\left(E < E_Q\right), \tag{15}$$

which we obtained earlier for the case of slowing-down in an infinite medium of hydrogen with no absorption.