[Slowing-down Theory]
(Chapter 1.9 in *Elements*)
A source of fast neutrons drives the moderated slow-neutron sources used in neutron scattering studies. Important for understanding their use are descriptions of the spectrum and time dependence of neutrons in a moderating medium (discussed further in Chapter 2 of *Elements*). The results are of special importance in relation to pulsed sources, in which slowing-down neutrons up to, say, 1 keV, are used in scattering measurements.

Here we treat the evolution of a pulse of fast neutrons as they slow down as functions of time and energy. We follow, in rough outline, the development according to Beckurts and Wirtz (1964). The time-dependent diffusion equation with a simple inelastic scattering kernel and a fast neutron source provides a useful example. "Inelastic scattering" in the present context refers to scattering events in the laboratory frame of reference, which are elastic in the center-of-mass frame and assumed to take place with a free nucleus initially at rest. This is a good approximation for epithermal neutrons but inappropriate for low-energy neutrons, which sense the effects of binding and thermal motions of the scattering atoms.

Time-dependent slowing-down equation

When a neutron with initial laboratory speed V_o collides with a nucleus of mass M initially at rest the velocity of the center of mass is

$$v_{com} = \frac{m}{m+M} V_o \qquad (1)$$

and the neutron velocity in the center-of-mass coordinate system (CMCS) is

$$v_{n\,com} = V_o - v_{com} = \frac{M}{m+M} V_o . \qquad (2)$$

After scattering at 180 deg in the CMCS, the laboratory speed of the scattered neutron is

$$v'_n(180°) = v_{com} - v_{n\,com} = \frac{(m-M)}{(m+M)} V_o , \qquad (3)$$

while the forward-scattered neutron (not scattered at all) speed is unchanged. The energy of the backscattered neutron, the minimum energy with which the neutron can emerge from the collision, is then

$$E_n(180°) = \frac{(m-M)^2}{(m+M)^2} E_{no} = \frac{(A-1)^2}{(A+1)^2} E_{no} , \qquad (4)$$

where $A = \frac{m}{M}$ is the mass number of the struck nucleus. When the scattering in the CMCS is isotropic (true for low-energy neutrons), the scattered energy distribution is constant for $\alpha E_o < E < E_o$ and the differential inelastic scattering cross section (assumed constant in space and for a monoisotopic material) has the form

$$\frac{\partial \Sigma_{scat}}{\partial E}\left(E_o \rightarrow E\right) = \frac{\Sigma_o}{\left(1-\alpha\right)E_o}\begin{cases} 1 \text{ for } \alpha E_o < E < E_o \text{ and} \\ 0 \text{ for } E < \alpha E_o \text{ or } E > E_o \end{cases} . \tag{5}$$

The differential inelastic scattering cross section vanishes for $E > E_o$ because we have assumed that the struck atom is initially at rest. The minimum relative final energy of the neutron, α, is

$$\alpha = \frac{\left(A-1\right)^2}{\left(A+1\right)^2} . \tag{6}$$

Clearly, α is always less than 1. The total scattering cross section is then

$$\Sigma_{scatt}\left(E_o\right) = \int_0^\infty \frac{\partial \Sigma_{scatt}}{\partial E}\left(E_o \rightarrow E\right) dE = \frac{\Sigma_o}{(1-\alpha)E_o}\int_{\alpha E_o}^{E_o} dE = \Sigma_o . \tag{7}$$

That is, the constant Σ_o in the expression for the inelastic scattering cross section is the total scattering cross section, $\Sigma_{scat}\left(E_o\right)$. For scattering from hydrogen, $\alpha = 0$: the neutron can lose all of its kinetic energy in a single, head-on collision, and the minimum energy after scattering is zero.

Because it is conventional in discussing the present subject, we now go over to the energy E rather than the speed v to represent the energy-related variable, using the same function names in spite of the risk of confusing functions having different arguments but distinguishing between them in terms of the argument names. (Tables and computers, which expect numerical values for the arguments of functions, can't do this—different functions must have different names.) The time-dependent diffusion equation is

$$\frac{1}{v(E)}\frac{\partial \varphi}{\partial t} = D(E)\nabla^2 \varphi(\vec{r},E,t) - \Sigma_{total}(E)\varphi(\vec{r},E,t) + \int_E^{\frac{E}{\alpha}} dE' \frac{\Sigma_{scatt}(E')}{\left(1-\alpha\right)E'}\varphi(\vec{r},E',t) + S(\vec{r},E,t) . \tag{8}$$

We can learn a great deal from this equation in special cases of interest in relation to how neutron sources work.