[Maxwell-Boltzmann]
(Chapter 1.11.2 in *Elements*)
Maxwell-Boltzmann distribution
The number density distribution of neutron speeds is

$$n(v)dv = \int_{4\pi} n\left(v\vec{\Omega}\right)v^2 d^2\Omega dv = \frac{4}{\sqrt{\pi}} N \frac{v^2}{v_T^3} \exp\left(-\frac{v^2}{v_T^2}\right) dv \quad . \tag{1}$$

The mean speed in the number density distribution is

$$\bar{v}_n = \frac{1}{N}\int_0^\infty v\, n(v)dv = \frac{2}{\sqrt{\pi}} v_T \quad . \tag{2}$$

The neutron flux with respect to any energy-related variable is $\varphi = nv$, and the flux per unit speed is

$$\varphi(v) = vn(v) = \frac{4N}{\sqrt{\pi}} \frac{v^3}{v_T^3} \exp\left(-\frac{v^2}{v_T^2}\right) \quad . \tag{3}$$

All are related through the jacobians elating the variables. The flux per unit energy is

$$\varphi_E(E) = \frac{vn(E)}{\frac{dE}{dv}} = \frac{2N}{\sqrt{\pi}} v_T \frac{E}{E_T^2} \exp\left(-\frac{E}{E_T}\right) = \varphi_T \frac{E}{E_T^2} \exp\left(-\frac{E}{E_T}\right) \; , \tag{4}$$

where φ_T is the integrated thermal neutron flux,

$$\varphi_T = N\bar{v}_n = \frac{2v_T}{\sqrt{\pi}} N \, . \tag{5}$$

Similarly, the flux per unit wavelength is

$$\varphi_\lambda(\lambda) = \varphi_E(E)\frac{dE}{d\lambda} = 2\varphi_T \frac{\lambda_T^4}{\lambda^5} \exp\left(-\frac{\lambda_T^2}{\lambda^2}\right) , \tag{6}$$

where

$$E = \frac{h^2}{2m\lambda^2} . \tag{7}$$

In practical cases with absorption and leakage and with general scattering cross sections that obey the principle of detailed balance with temperature *T*, the neutron spectrum is not really a Maxwellian distribution with temperature *T*. Rather; it is the solution of the energy-dependent balance equation.

A deeply theoretical comment

However, the spectrum really consists of a sum of Maxwellian-like functions, the energy eigenfunctions of the energy-dependent balance equation, in which the predominant term is usually the effective-temperature Maxwellian. Further discussion involves rather deep consideration of the eigenvalue spectrum of the balance equation, which we cannot deal with here. However, Williams (1969, 125 ff) relates that spectral functions in media in which the minimum scattering cross section occurs at high energies—which are the most common moderators in neutron sources—are distinctly different from those in which there is a minimum in the scattering cross section in the range ~ $k_B T$. Evident at the time of his writing were the cases of polycrystalline and liquid coherently scattering materials, which exhibit a minimum cross section for energies just below the Bragg cutoff energy (e.g., D_2, D_2O, Be, C, O_2, …). Current neutron sources employ liquid hydrogen moderators in which para-hydrogen predominates and for which the total scattering cross section exhibits a strong minimum at about 15 meV. Liquid helium also has such a low-energy minimum below approximately 1 meV. At this time, little is known about the relationship of the energy eigenfunctions and time eigenvalues to the existence of these cross section minima. However, as needs for cold and very cold neutrons continue to evolve, so must these questions receive greater attention: theoretical, experimental, and calculational. Furthermore, these considerations may have significant bearing on functional representations used for characterizing moderators, as in Chapter 2.3 of *Elements*.