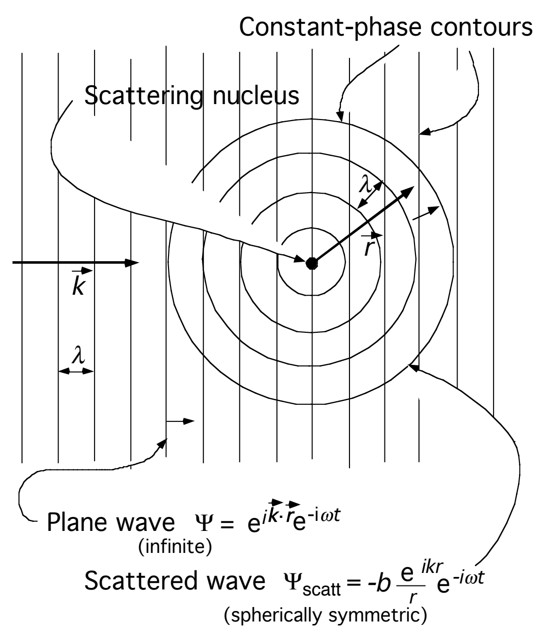
[Bound-Atom Scattering]
(Chapter 1.3.2 in *Elements*)
There is a fundamental conceptual distinction between bound-atom and free-atom neutron-nuclear scattering interactions. In the quantum mechanical description of the neutron-nucleus scattering, the neutron wave scattered from a single nucleus rigidly fixed (or infinitely massive) at the origin of coordinates is spherically symmetric,

$$\Psi_{scatt} = -\,b\left(\frac{\exp\,(ikr)}{r}\right)\exp\,(-\,i\omega t), \qquad (1)$$

in which b is the *bound-atom scattering length* and is related to the constant a in the Fermi pseudopotential,

$$V(\vec{\rho}) = \frac{2\pi\hbar^2}{m} b\delta\left(\vec{\rho}\right), \quad b = \frac{m}{2\pi\hbar^2}a. \qquad (2)$$

The figure illustrates a plane wave of infinite extent and a spherically symmetric outgoing scattered wave, which the reader can demonstrate is a solution of the wave equation in the region where $V(\vec{r}) = 0$. *Constant-phase contours* represent loci on which, for example, $\vec{k}\cdot\vec{r} - \omega t = constant$, so that on a contour $|\vec{r}|$ increases in the direction of $\vec{k}$ as time t increases.



An incident plane wave of infinite extent and a spherically symmetric scattered wave. The diagram greatly exaggerates the size of the scattering nucleus in relation to the wavelength of slow neutrons. Small arrows indicate the direction of wave propagation.

When the incident wave is a plane wave as in the figure and when the amplitude of the scattered wave is the scattering length b, then the ratio of the scattering rate to the incident current density $\frac{I_{out}}{J_{in}}$ is the definition of the cross section for scattering from a single nucleus.
The scattered-wave wave function represents a total outgoing current

$$I_{out} = \lim_{r \to \infty} \int_{4\pi} \vec{r} \cdot \vec{J}_{out} r^2 d\Omega = 4\pi \left(\frac{\hbar}{m} \right) k b^2 = 4\pi v b^2 , \qquad (3)$$

while the incoming wave function represents an incoming current density

$$J_{in} = \left| \vec{J} \right| = v. \qquad (4)$$

The scattering cross section for the present case is

$$\sigma_{scatt} = \frac{I_{out}}{J_{in}} = 4\pi b^2, \qquad (5)$$

which has the dimensions of an area, effectively the area that the nucleus presents to the incident beam. Because we have treated the scattering from a fixed nucleus (a bound atom), in effect having infinite mass, this is the *bound-atom scattering cross section.*

The distinction between bound- and free-atom cases lies entirely in the choice of coordinate system in which to describe the two-particle system. Equation 3 represents the struck particle as the center of the interaction, therefore effectively of infinite mass: a bound atom. In the free-atom case, the Schrödinger equation is of the same form but the coordinate is the distance between particles and the mass is the reduced mass of the neutron-nuclear system. The c-o-m coordinate is related to the laboratory coordinate by the usual transformation involving the masses and initial velocities of the particles, taking the struck particle to be at rest (further explained in Chapter 3.4.2 in *Elements*).