[Angular Distribution]
(Chapter 1.12 of *Elements*)
Angular distribution of the neutrons at a vacuum interface
Using the *Fermi approximation*, we find that the total outgoing current in terms of the flux is

$$J_{out} = \int_{4\pi} \mu\psi_o(\mu) d^2\Omega = \frac{1}{\sqrt{3}}\varphi_o \tag{1}$$

and the scalar flux at the surface is related to the forward-directed angular current density as

$$\varphi_o = 2\pi \frac{\left(1 + \sqrt{3}/2\right)}{\left(1 + \sqrt{3}\right)} \psi_o\left(\mu = 1\right) = 4.29\psi_o\left(\mu = 1\right). \tag{2}$$

This last result is useful for relating measurements made at a distance from the moderator to the scalar flux at the surface of the source.

The P_1 approximation
So-called because it represents the angular distribution as an expansion in Legendre functions only up to first order, the *P_1 approximation* to the neutron transport equation represents the angular variation of the neutron distribution as the sum of two terms (now ignoring the energy and time dependence),

$$\varphi\left(\vec{\boldsymbol{r}}, \vec{\Omega}, v\right) = \varphi_o\left(\vec{\boldsymbol{r}}, v\right) + \varphi_1\left(\vec{\boldsymbol{r}}, v\right)\vec{\Omega} \quad , \tag{3}$$

that is, an isotropic term and a term linear in the direction vector $\vec{\Omega}$,

$$\varphi\left(\vec{\boldsymbol{r}}, \vec{\Omega}\right) = \frac{1}{4\pi}\varphi_o(\vec{\boldsymbol{r}}) + \frac{3}{4\pi}\vec{J}(\vec{\boldsymbol{r}})\cdot\vec{\Omega} \quad . \tag{4}$$

Here, $\varphi_o(\vec{\boldsymbol{r}})$ is the scalar flux and $\vec{J}(\vec{\boldsymbol{r}})$ is the net angular current, a vector quantity. The P_1 theory provides an approximation (see Duderstadt and Hamilton [1976, 142 ff]) to the angular distribution at the surface in which the net reentering current is zero, but it is physically unrealistic—the theory is good only in the case of mild anisotropy and distant from vacuum interfaces. For the case of the angular distribution at a planar surface, $\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_s$, symmetry demands that the current vector is parallel to the normal vector,

$$\vec{J}(\vec{\boldsymbol{r}}_s) = J(\vec{\boldsymbol{r}}_s)\vec{\boldsymbol{n}} \tag{5}$$

and the no-reentrant-current condition is

$$\int_{\vec{\boldsymbol{n}}\cdot\vec{\Omega} < 0} \vec{\boldsymbol{n}}\cdot\vec{\Omega}\varphi\left(\vec{\boldsymbol{r}}, \vec{\Omega}\right) d^2\vec{\Omega} = -\frac{1}{4}\varphi_o(\vec{\boldsymbol{r}}_s) + \frac{1}{2}J(\vec{\boldsymbol{r}}_s) = 0 \quad , \tag{6}$$

so that the angular current density at the surface is

$$\psi(\mu) = \frac{1}{4\pi}\varphi_o\left(1 + \frac{3}{2}\mu\right) . \tag{7}$$

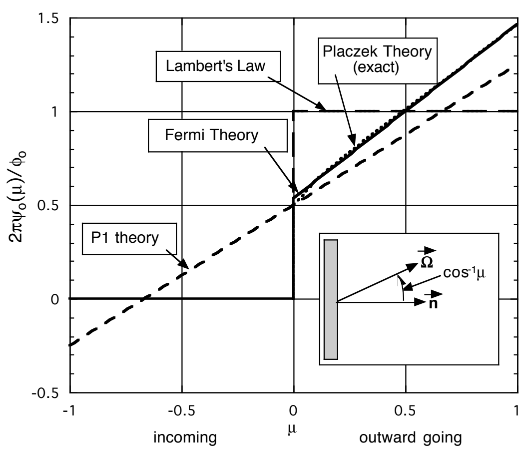
The P_1 theory applied to the spatial distribution of flux near the boundary gives the extrapolated boundary condition discussed in [Thermal-Neutron Spatial Distribution].

Lambert's Law

The well-known *Lambert's Cosine Law* (pronouncd as French) approximates the angular distribution as simply proportional to the projected area of the emitting surface in the direction of outward-directed rays, $\mu = \cos(\theta)$, on the basic assumption that the angular current density is constant in angle,

$$\psi(\mu) = \frac{\varphi_o}{4\pi}\mu . \tag{8}$$

which turns out not to be very realistic in the case of neutron transport. The following figure shows the several approximations. Fermi's approximation should be sufficient for most purposes.



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