[dipole-dipole magnetic scattering] Derivation of the double-differential cross section of magnetic scattering

Here, we point out the key steps by which (4.25), the double-differential cross section of magnetic scattering, is derived from (4.24), the matrix element of the dipole-dipole interaction of the neutron's magnetic moment with the electronic spin and orbital currents, in {§4.2:145}. The matrix element is

$$\left|\left\langle \vec{\mathbf{k}}_1\varepsilon_f\right|\sum_e\left[\vec{\sigma}\cdot\nabla\times\frac{\vec{\mathbf{s}}_e\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}_e)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_e|^3}-\frac{1}{2\hbar}\left(\vec{\mathbf{p}}_e\cdot\frac{\vec{\sigma}\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}_e)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_e|^3}+\frac{\vec{\sigma}\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}_e)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_e|^3}\cdot\vec{\mathbf{p}}_e\right)\right]\left|\vec{\mathbf{k}}_0\varepsilon_i\right\rangle\right|^2, \quad (1)$$

where the first and second terms in the square bracket represent the neutron dipole-electron dipole interaction and the neutron spin-electron orbital motion interaction, respectively.

Noting $\frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|^3}=-\vec{\nabla}\left(\frac{1}{|\vec{\mathbf{R}}|}\right)=-\vec{\nabla}\left(\frac{1}{2\pi^2}\int d^3\vec{\mathbf{q}}\frac{1}{q^2}\exp\left(i\vec{\mathbf{q}}\cdot\vec{\mathbf{R}}\right)\right)$, we have

$$\begin{aligned}-\vec{\nabla}\left(\vec{\mathbf{s}}_e\times\vec{\nabla}\right)\left(\frac{1}{|\vec{\mathbf{R}}|}\right)&=-\frac{1}{2\pi^2}\int d^3\vec{\mathbf{q}}\frac{1}{q^2}\left(\vec{\nabla}\times\vec{\mathbf{s}}_e\times\vec{\nabla}\right)\exp\left(i\vec{\mathbf{q}}\cdot\vec{\mathbf{R}}\right)\\&=\frac{1}{2\pi^2}\int d^3\vec{\mathbf{q}}\frac{1}{q^2}\left[\vec{\mathbf{q}}\times\left(\vec{\mathbf{s}}_e\times\vec{\mathbf{q}}\right)\right]\exp\left(i\vec{\mathbf{q}}\cdot\vec{\mathbf{R}}\right).\end{aligned} \quad (2)$$

Likewise, $\frac{2\pi\hbar^2}{m}\left\langle\vec{\mathbf{k}}_1\left|\vec{\mathbf{p}}_e\cdot\frac{\vec{\sigma}\times\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|^3}\right|\vec{\mathbf{k}}_0\right\rangle=-\frac{4\pi i}{|\vec{\mathbf{Q}}|}\exp\left(i\vec{\mathbf{Q}}\cdot\vec{\mathbf{R}}\right)\vec{\sigma}\cdot\left(\tilde{\mathbf{Q}}\times\vec{\mathbf{p}}_e\right),$ (3)

where

$$\vec{\mathbf{Q}}=\vec{\mathbf{k}}_0-\vec{\mathbf{k}}_1, \quad \text{and} \quad \hat{\mathbf{Q}}=\frac{\vec{\mathbf{Q}}}{|\vec{\mathbf{Q}}|}. \quad (4)$$

And from (2),

$$\frac{2\pi\hbar^2}{m}\left\langle\vec{\mathbf{k}}_1\left|\vec{\sigma}\cdot\vec{\nabla}\times\frac{\vec{\mathbf{s}}_e\times\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|^3}\right|\vec{\mathbf{k}}_0\right\rangle=4\pi\exp\left(i\vec{\mathbf{Q}}\cdot\vec{\mathbf{R}}\right)\left[\hat{\mathbf{Q}}\times\left(\vec{\mathbf{s}}_e\times\hat{\mathbf{Q}}\right)\right]. \quad (5)$$

Finally, combining (3) and (5) into (4.24) and using

$$\vec{\mathbf{M}}_\perp\left(\vec{\mathbf{Q}}\right)=\sum_e\exp\left(i\vec{\mathbf{Q}}\cdot\vec{\mathbf{r}}_e\right)\left[\hat{\mathbf{Q}}\times\vec{\mathbf{s}}_e\times\hat{\mathbf{Q}}+\frac{i}{\hbar Q}\left(\vec{\mathbf{p}}_e\times\hat{\mathbf{Q}}\right)\right], \quad (4.30)$$

we arrive at

$$\left(\frac{d^2\sigma}{d\Omega dE_1}\right)_{mag} = (\gamma r_0)^2 \frac{k_1}{k_0} \sum_{i,f} p(\varepsilon_i) g(\varepsilon_f) \left\langle \varepsilon_i \middle| \left(\vec{\sigma}\cdot\vec{\mathbf{M}}_\perp\right)^+ \middle| \varepsilon_f \right\rangle \left\langle \varepsilon_f \middle| \vec{\sigma}\cdot\vec{\mathbf{M}}_\perp \middle| \varepsilon_i \right\rangle \delta\left(E + E_{\varepsilon_0} - E_{\varepsilon_1}\right). \quad (4.25)$$

[Note: in the book, Eq. (4.30) is named twice for two different equations. The correct numbering should be: (4.29) → (4.29a), and the first (4.30) → (4.29b)]

Reference:

Lovesey, S. W. (1984), “Theory of Neutron Scattering from Condensed Matter”, Vol. 2. Clarendon Press (Oxford, UK), §7.2.