[Breit-Wigner] The Breit-Wigner formula for isolated resonances

The optical model in general does not address the resonances in the observed cross sections because its effective neutron-nucleus potential provides no information regarding the internal structure of the compound nucleus (and we don't have a rigorous theory to predict it!) {§3.1:94}. The Breit-Wigner formula provides a convenient way to describe a resonance in terms of its energy and half-width. It is well suited for isolated resonances where the observed widths are small compared to the energy spacing of the resonances (plenty of such resonances in the Barn Book). Although a general formula can be obtained for all *l*-waves, for brevity we only present the case of $l = 0$.

From (3.16), we have

$$r\psi(r) = e^{-ik_0 r} - \eta_0 e^{ik_0 r}, \tag{1}$$

so that

$$f(E) = R\left[\frac{\frac{d}{dr}(r\psi)}{r\psi}\right]_{r=R} = -ik_0 R \frac{1+\eta_0 e^{2ik_o R}}{1-\eta_0 e^{2ik_o R}} \equiv f_0 - ig, \tag{2}$$

where E is the neutron energy, R is the boundary of the neutron-nucleus interaction range, $(0, R)$, and η_0 is the *s*-wave *reflection factor* or *elasticity parameter*, see (3.21). Here, (2) is just a trick to express the unknown as the "logarithmic derivative of the radial wavefunction at the nuclear boundary" and in terms of parameters f_0 and g. Now solving η_0 and substituting it into the cross sections of (3.22) and (3.23), we find the elastic-scattering and reaction cross sections for $l = 0$ as:

$$\sigma_{0,el} = \frac{4\pi}{k_0^2}\left|\frac{k_0 R}{i(k_0 R + g) - f_0} + e^{ik_0 R}\sin(k_0 R)\right|^2, \tag{3}$$

and

$$\sigma_{0,r} = \frac{4\pi}{k_0^2}\frac{k_0 R g}{(k_0 R + g)^2 + f_0^2}., \tag{4}$$

The cross sections reach a maximum value if $f_0(E) = 0$ for $E = E_{res}$, which correspond to a resonance with a *resonance energy* of E_{res}. Next, expanding $f_0(E)$ in power series of $E - E_{res}$ and keeping only the first-order term, we obtain

$$f_0(E) \cong \left(\frac{\partial f}{\partial E}\right)_{E=E_{res}} (E - E_{res}). \tag{5}$$

Then we may invoke a connection of the unknown parameters to the experimentally observable widths, i.e., the *neutron width* Γ_e and the *reaction width* Γ_r as:

$$\Gamma_e \equiv -\frac{2k_0 R}{\left[\partial f/\partial E\right]_{E=E_{res}}}, \tag{6}$$

$$\Gamma_r \equiv -\frac{2g}{\left[\partial f / \partial E\right]_{E=E_{res}}}. \tag{7}$$

Finally, we obtain

$$\sigma_{0,r} = \frac{\pi}{k_0^2} \frac{\Gamma_r \Gamma_e}{\left(E - E_{res}\right)^2 + \Gamma^2 / 4}, \tag{8}$$

where

$$\Gamma \equiv \Gamma_e + \Gamma_r \tag{9}$$

is the *total width* of the resonance. It can be shown that the *l*-wave elastic and reaction cross sections of the isolated resonance are:

$$\sigma_{l,el} = \frac{\pi}{k_0^2}(2l+1) \frac{\Gamma_e^2}{\left(E - E_{res}\right)^2 + \Gamma^2 / 4}, \tag{10}$$

$$\sigma_{l,r} = \frac{\pi}{k_0^2}(2l+1) \frac{\Gamma_e \Gamma_r}{\left(E - E_{res}\right)^2 + \Gamma^2 / 4}, \tag{11}$$

Eqs. (8) and (10-11) are called the *Breit-Wigner formulas*.